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Physics Letters B

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Reconstruction of scalar field theories realizing inflation consistent with the Planck and BICEP2 results

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ARTICLE INFO

Article history:

Received 10 June 2014

Received in revised form 4 September 2014

Accepted 4 September 2014

Available online 10 September 2014

Editor: J. Hisano

ABSTRACT

We reconstruct scalar field theories to realize inflation compatible with the BICEP2 result as well as the Planck. In particular, we examine the chaotic inflation model, natural (or axion) inflation model, and an inflationary model with a hyperbolic inflaton potential. We perform an explicit approach to find out a scalar field model of inflation in which any observations can be explained in principle.

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1. Introduction

In addition to cosmological observations to assist the current accelerated expansion of the universe, the so-called dark energy problem (for recent reviews, see [1,2]), thanks to the observational data obtained from the recent BICEP2 experiment [3], the Planck satellite [4,5] as well as the Wilkinson Microwave anisotropy probe (WMAP) [6,7], the studies on inflation in the early universe have recently been executed much more extensively.

In particular, there have been proposed inflationary models, especially, scalar field theories, to fit the observational data obtained from the BICEP2 experiment [3] as well as the Planck satellite [4,5] including the running of the spectral index for the scalar modes of the primordial density perturbations in Refs. [8,9]¹ (for a review on the relation between the form of the inflaton potential and the power spectrum of the density perturbations, see [13]).

In this Letter, we reconstruct the scalar field theories to explain the BICEP2 and Planck data. Very recently, conditions that a model is consistent with the BICEP2 result have been examined in Refs. [14–24]. Our aim in this work is to reconstruct wider

classes of scalar field theories with inflation. It is also checked whether indeed the running of the spectral index exists and the models reconstructed may be compatible with the BICEP and Planck data. Particularly, we investigate the so-called chaotic inflation model [25], natural (or recently called as axion) inflation model [26], and an inflationary model where the inflaton has a hyperbolic potential, which was first proposed in Ref. [27]. In this way, we can provide the method to build the scalar field models fitting any data of future observations different from the BICEP and Planck. We use units of $k_B = c = \hbar = 1$ and express the gravitational constant $8\pi G$ by $\kappa^2 \equiv 8\pi/M_{\text{Pl}}^2$ with the Planck mass of $M_{\text{Pl}} = G^{-1/2} = 1.2 \times 10^{19}$ GeV.

The Letter is organized as follows. In Section 2, we describe the reconstruction of the scalar field theory. In Section 3, we present the scalar field models compatible with the Planck and BICEP2 data. Summary is finally presented in Section 4.

2. Reconstruction of scalar field theories

We study the model of a scalar field coupled with gravity

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right). \quad (2.1)$$

The slow-roll parameters ϵ , η and ξ are given by

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¹ For an approach to the model building based on the modification of gravity, see, e.g., [10–12].

$$\begin{aligned}\epsilon &\equiv \frac{1}{2\kappa^2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2, & \eta &\equiv \frac{1}{\kappa^2} \frac{V''(\phi)}{V(\phi)}, \\ \xi^2 &\equiv \frac{1}{\kappa^4} \frac{V'(\phi)V'''(\phi)}{V(\phi)^2}.\end{aligned}\quad (2.2)$$

In the following, the prime denotes the derivative with respect to the argument of functions such as $V'(\phi) \equiv \partial V(\phi)/\partial \phi$. The tensor-to-scalar ratio is obtained as

$$r = 16\epsilon, \quad (2.3)$$

and the spectral index n_s of the primordial curvature fluctuations and the associated running of the spectral index α_s are

$$n_s - 1 \sim -6\epsilon + 2\eta, \quad \alpha_s \equiv \frac{dn_s}{d \ln k} \sim 16\epsilon\eta - 24\epsilon^2 - 2\xi^2. \quad (2.4)$$

For a detailed review of inflation including the expiations of the above slow-roll parameters, see, for instance, [28]. The number of e -folds N or the redshift z dependence or the slow-roll parameter ϵ or the tensor-to-scalar ratio r will be determined by the future observations. Then we now explore how we can construct a model which reproduces the observed values of the slow-roll parameter ϵ or the tensor-to-scalar ratio r .

We take the flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric $ds^2 = -dt^2 + a^2(t) \sum_{i=1,2,3} (dx^i)^2$. Here, $a(t)$ is the scale factor and the Hubble parameter is defined as $H \equiv \dot{a}/a$ with the dot showing the time derivative.

The gravitational field equations in the FLRW background are given by

$$\frac{3}{\kappa^2} H^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad -\frac{1}{\kappa^2} (3H^2 + 2\dot{H}) = \frac{1}{2} \dot{\phi}^2 - V(\phi). \quad (2.5)$$

We may redefine the scalar field ϕ by a new scalar field φ , $\phi = \phi(\varphi)$, and identify φ as the number of e -folds N . Hence, gravitational field equations (2.5) can be written as

$$\begin{aligned}\frac{3}{\kappa^2} H(N)^2 &= \frac{1}{2} \omega(\varphi) H(N)^2 + V(\phi(\varphi)), \\ -\frac{1}{\kappa^2} (3H(N)^2 + 2H'(N)H(N)) &= \frac{1}{2} \omega(\varphi) H(N)^2 - V(\phi(\varphi)),\end{aligned}\quad (2.6)$$

with $\omega(\varphi) \equiv (d\phi/d\varphi)^2$. It is mentioned that the reconstruction of such a class of models yielding the equations in (2.6) was first suggested in Ref. [29]. Thus, if the Hubble parameter H is represented as a function of N as $H(N)$, and if $\omega(\varphi)$ and $V(\varphi) \equiv V(\phi(\varphi))$ become

$$\begin{aligned}\omega(\varphi) &= -\frac{2H'(N)}{\kappa^2 H(N)} \Big|_{N=\varphi}, \\ V(\varphi) &= \frac{1}{\kappa^2} (3H(N)^2 + H(N)H'(N)) \Big|_{N=\varphi},\end{aligned}\quad (2.7)$$

we have $H = H(N)$, $\varphi = N$ as a solution for the equation of motion of ϕ or φ and the Einstein equation. We should note $H' < 0$ because $\omega(\varphi) > 0$.

We here express the slow-roll parameters ϵ , η and ξ by H as follows

$$\begin{aligned}\epsilon &= \frac{1}{2\kappa^2} \left(\frac{d\phi}{d\varphi} \right)^2 \left(\frac{V'(\varphi)}{V(\varphi)} \right)^2 \Big|_{\varphi=N} = \frac{1}{2\kappa^2} \frac{1}{\omega(\varphi)} \left(\frac{V'(\varphi)}{V(\varphi)} \right)^2 \Big|_{\varphi=N} \\ &= -\frac{H(N)}{4H'(N)} \left[\frac{6\frac{H'(N)}{H(N)} + \frac{H''(N)}{H(N)} + (\frac{H'(N)}{H(N)})^2}{3 + \frac{H'(N)}{H(N)}} \right]^2,\end{aligned}$$

$$\begin{aligned}\eta &= \frac{1}{\kappa^2 V(\varphi)} \left[\frac{d\phi}{d\varphi} \frac{d}{d\varphi} \left(\frac{d\phi}{d\varphi} \right) V'(\varphi) + \left(\frac{d\phi}{d\varphi} \right)^2 V''(\varphi) \right] \Big|_{\varphi=N} \\ &= \frac{1}{\kappa^2 V(\varphi)} \left[-\frac{\omega'(\varphi)}{2\omega(\varphi)^2} V'(\varphi) + \frac{1}{\omega(\varphi)} V''(\varphi) \right] \Big|_{\varphi=N} \\ &= -\frac{1}{2} \left(3 + \frac{H'(N)}{H(N)} \right)^{-1} \left[9 \frac{H'(N)}{H(N)} + 3 \frac{H''(N)}{H(N)} + \frac{1}{2} \left(\frac{H'(N)}{H(N)} \right)^2 \right. \\ &\quad \left. - \frac{1}{2} \left(\frac{H''(N)}{H'(N)} \right)^2 + 3 \frac{H''(N)}{H'(N)} + \frac{H'''(N)}{H'(N)} \right], \\ \xi^2 &= \frac{V'(\varphi)}{\kappa^4 V(\varphi)^2 \omega(\varphi)^2} \left\{ \left[-\frac{\omega'(\varphi)}{2\omega(\varphi)} + \left(\frac{\omega'(\varphi)}{\omega(\varphi)} \right)^2 \right] V'(\varphi) \right. \\ &\quad \left. - \frac{3\omega'(\varphi)}{2\omega(\varphi)} V''(\varphi) + V'''(\varphi) \right\} \Big|_{\varphi=N} \\ &= \frac{6\frac{H'(N)}{H(N)} + \frac{H''(N)}{H(N)} + (\frac{H'(N)}{H(N)})^2}{4(3 + \frac{H'(N)}{H(N)})^2} \left[3 \frac{H(N)H'''(N)}{H'(N)^2} + 9 \frac{H'(N)}{H(N)} \right. \\ &\quad \left. - 2 \frac{H(N)H''(N)H'''(N)}{H'(N)^3} + 4 \frac{H''(N)}{H(N)} \right. \\ &\quad \left. + \frac{H(N)H''(N)^3}{H'(N)^4} + 5 \frac{H'''(N)}{H'(N)} - 3 \frac{H(N)H''(N)^2}{H'(N)^3} \right. \\ &\quad \left. - \left(\frac{H''(N)}{H'(N)} \right)^2 + 15 \frac{H''(N)}{H'(N)} + \frac{H(N)H''''(N)}{H'(N)^2} \right].\end{aligned}\quad (2.8)$$

When we solve equations in (2.8) with respect to $H(N)$, we can find the corresponding scalar field theory by using (2.7), in principle. It is not so straightforward to solve equations in (2.8). Therefore, we investigate the following case

$$H'''(N) \ll H''(N) \ll H'(N) \ll H(N). \quad (2.9)$$

In this case, we acquire

$$\epsilon(N) \sim -\frac{H'(N)}{H(N)} \quad \text{or} \quad H(N) \sim H_0 \exp \left(-\int d\hat{N} \epsilon(\hat{N}) \right), \quad (2.10)$$

with a constant H_0 and also

$$\eta(N) \sim -\frac{3}{2} \frac{H'(N)}{H(N)} \sim \frac{3}{2} \epsilon(N), \quad \xi^2 \sim \frac{3}{2} \left(\frac{H'(N)}{H(N)} \right)^2 \sim \frac{3}{2} \epsilon^2. \quad (2.11)$$

Thus, by using (2.8), we have

$$\begin{aligned}\omega(\varphi) &\sim \frac{2}{\kappa^2} \epsilon(N) \Big|_{N=\varphi}, \\ V(\varphi) &\sim \frac{3H_0^2}{\kappa^2} \exp \left(-2 \int d\hat{N} \epsilon(\hat{N}) \right) \Big|_{N=\varphi}.\end{aligned}\quad (2.12)$$

Furthermore, by combining the relations in (2.4) with those in (2.10) and (2.11), we find

$$n_s - 1 \sim -3\epsilon, \quad \alpha_s \sim -3 \left(\frac{H'(N)}{H(N)} \right)^2 \sim -3\epsilon^2. \quad (2.13)$$

We should note that the running of the spectral index α_s is not always small and negative.

3. Scalar field theories compatible with the Planck and BICEP2 results

We study a scalar field theory consistent with the BICEP2 result and that compatible with the Planck data in terms of the tensor-to-scalar ratio r by taking account of the running of the spectral index α_s .

According to the Planck analysis [4,5], $n_s = 0.9603 \pm 0.0073$ (68%CL) and $\alpha_s = -0.0134 \pm 0.0090$ (68%CL) with the Planck and WMAP [6,7] data. The sign of α_s is negative at 1.5σ level. Furthermore, $r < 0.11$ (95%CL). On the other hand, the result of the BICEP2 experiment is $r = 0.20^{+0.07}_{-0.05}$ (68%CL) [3] (for very recent discussions on the data analysis of the foreground data related to the BICEP2 result, see, e.g., Refs. [30,31]).

We here remark that from the consequences of the reconstruction in (2.13) with Eq. (2.13), e.g., for $\epsilon = 1.1 \times 10^{-2}$, we acquire $n_s = 0.967$, $r = 0.176$, and $\alpha_s = -3.63 \times 10^{-4}$. Hence, it is interpreted that the reconstruction procedure could lead to the scalar field theories realizing inflation compatible with the Planck and BICEP2 results.

In inflationary models with a single inflaton field ϕ , the Lyth bound [32] has been known regarding the difference of the inflation amplitude ϕ_i at the initial time t_i of inflation from that ϕ_f at the final time t_f of it. It can be represented as $\Delta(\kappa\phi) \equiv |\kappa\phi_i - \kappa\phi_f| \gtrsim N\sqrt{r/8}$ [14]. For $N = 50$ (60) and $r = 0.10$, we have $\Delta(\kappa\phi) \gtrsim 5.6$ (6.1) [33]. When $r \geq [<] 3.2$ (2.0) $\times 10^{-3}$ with $N = 50$ (60), we find $\Delta(\kappa\phi) \geq [<] 1$. Consequently, in what follows, as the condition for an inflation model consistent with the BICEP2 result, we consider $\Delta(\kappa\phi) \geq 1$, whereas as that compatible with the Planck data, we regard $\Delta(\kappa\phi) < 1$.

3.1. Inflationary model consistent with the BICEP2 result

First, we explore the so-called chaotic inflation model [25], where the inflaton potential of a canonical scalar field ϕ is given by

$$V(\phi) = \bar{V}_c (\kappa\phi)^p, \quad (3.1)$$

with \bar{V}_c and p constants. In this model, from equations in (2.2) we obtain

$$\epsilon \simeq \frac{p}{4N+p}, \quad \eta \simeq \frac{2(p-1)}{4N+p}, \quad \xi^2 \simeq \frac{4(p-1)(p-2)}{(4N+p)^2}. \quad (3.2)$$

In deriving these relation, we have used $\kappa\phi \simeq \sqrt{p(4N+p)/2}$. This comes from the condition of $\epsilon(\phi_f) < 1$, leading to $\phi > p/(\sqrt{2}\kappa)$, and the relation $N = -\int_{t_f}^t H(\hat{t})d\hat{t} \approx \kappa^2 \int_{\phi_f}^{\phi} (V(\hat{\phi})/V'(\hat{\phi}))d\hat{\phi}$, where the approximate equality follows from the relation $H \simeq -\kappa(V(\phi)/V'(\phi))(\kappa\dot{\phi})$ under the slow-roll approximation.

With Eq. (2.3) and the equations in (2.4), we acquire

$$n_s \simeq \frac{4(N-1)-p}{4N+p}, \quad r \simeq \frac{16p}{4N+p}, \quad \alpha_s \simeq -\frac{8(p+2)}{(4N+p)^2}. \quad (3.3)$$

For instance, when $(N, p) = (50, 2)$ and $(60, 3)$, we get $(n_s, r, \alpha_s) = (0.960, 0.158, -7.84 \times 10^{-4})$ and $(0.959, 0.198, -6.77 \times 10^{-4})$, respectively. Accordingly, this model could be considered to be consistent with the BICEP result, even though the absolute value of α_s is smaller than that of the Planck one.

3.2. Inflationary model compatible with the Planck analysis

Next, we investigate the so-called natural (or axion) inflation model [26] with the following inflaton potential of a pseudo-scalar field ϕ such as a pseudo-Nambu–Goldstone boson

$$V(\phi) = \bar{V}_a \left[1 + \cos\left(\frac{\phi}{f_a}\right) \right], \quad (3.4)$$

where \bar{V}_a and f_a are constants, and ϕ/f_a is a dimensionless combination. The number of e -folds could be represented as [34]

$$N = 2\kappa^2 f_a^2 \ln \left[\frac{\sin(\phi_f/f_a)}{\sin(\phi_h/f_a)} \right], \quad (3.5)$$

where ϕ_h is the amplitude of ϕ at the time t_h when the curvature perturbation with the wave number k first crosses the horizon during inflation. In this case, N is considered to be the value of the number of e -folds for the curvature perturbation with the wave number k . Moreover, from a relation $\epsilon(\phi_f) = 1$ at the end of inflation, we see that ϕ_f/f_a is described as [34]

$$\frac{\phi_f}{f_a} = \arccos \left[\frac{1 - 2(\kappa f_a)^2}{1 + 2(\kappa f_a)^2} \right]. \quad (3.6)$$

For this model, it follows from equations in (2.2) that

$$\epsilon = \frac{1}{2\kappa^2 f_a^2} \frac{\sin^2(\phi_h/f_a)}{[1 + \cos(\phi_h/f_a)]^2}, \quad (3.7)$$

$$\eta = -\frac{1}{\kappa^2 f_a^2} \frac{\cos(\phi_h/f_a)}{1 + \cos(\phi_h/f_a)}, \quad (3.8)$$

$$\xi^2 = -\frac{1}{\kappa^4 f_a^4} \frac{\sin^2(\phi_h/f_a)}{[1 + \cos(\phi_h/f_a)]^2}, \quad (3.9)$$

where ϕ_h is described by Eq. (3.5) with N and ϕ_f as $\sin(\phi_h/f_a) = \sin(\phi_f/f_a) \exp[-N/(2\kappa^2 f_a^2)]$. By using this equation, Eq. (2.3), and equations in (2.4), we find

$$n_s = 1 - \frac{1}{\kappa^2 f_a^2} \frac{1}{[1 + \cos(\phi_h/f_a)]^2} \times \left\{ \sin^2\left(\frac{\phi_h}{f_a}\right) + 2 \left[1 + \cos\left(\frac{\phi_h}{f_a}\right) \right] \right\}, \quad (3.10)$$

$$r = \frac{8}{\kappa^2 f_a^2} \frac{\sin^2(\phi_h/f_a)}{[1 + \cos(\phi_h/f_a)]^2}, \quad (3.11)$$

$$\alpha_s = -\frac{4}{\kappa^4 f_a^4} \frac{\sin^2(\phi_h/f_a)}{[1 + \cos(\phi_h/f_a)]^3}. \quad (3.12)$$

The amplitude of ϕ at the first horizon crossing time for the number of e -folds $N = 50$ – 60 before the end of inflation is observed in the fluctuations in terms of the cosmic microwave background (CMB) radiation. As an example, for $(N, \kappa f_a) = (50.0, 5.00)$ and $(60.0, 5.50)$, we have $(n_s, r, \alpha_s) = (0.960, 8.37 \times 10^{-4}, -8.39 \times 10^{-6})$ and $(0.967, 5.85 \times 10^{-4}, -4.84 \times 10^{-6})$, respectively. In evaluating these numerical values, we have also used the related equations $\cos(\phi_h/f_a) = \sqrt{1 - \sin^2(\phi_f/f_a) \exp[-N/(\kappa^2 f_a^2)]}$ and $\sin(\phi_h/f_a) = 2\sqrt{2}\kappa f_a/[1 + 2(\kappa f_a)^2]$ following from Eq. (3.6). Consequently, this model can be regarded to be compatible with the Planck result.

3.3. Model with a hyperbolic inflaton potential

Furthermore, as a model of inflation, we explore the action of a scalar field theory in (2.1) with the following potential of the inflaton ϕ [27]:

$$V(\phi) = \bar{V}_n \cosh(\gamma_n \kappa \phi), \quad (3.13)$$

where \bar{V}_n and γ_n are constants. In this model, the number of e -folds during inflation reads

$$N = - \int_{t_f}^t H(\hat{t}) d\hat{t} \simeq \kappa^2 \int_{\phi_f}^{\phi} \frac{V(\hat{\phi})}{V'(\hat{\phi})} d\hat{\phi} \approx \ln \left[\frac{(\gamma_n \kappa \phi)}{(\gamma_n \kappa \phi_f)} \right], \quad (3.14)$$

where in deriving the last approximate equality, we have used $\gamma_n \kappa \phi \ll 1$ with $\gamma_n \ll 1$. In what follows, we concentrate on this regime. We remark that for $\gamma_n \kappa \phi \ll 1$, the inflaton potential in Eq. (3.13) becomes $V(\phi) \approx \bar{V}_n [1 + (1/2)(\gamma_n \kappa \phi)^2]$. Such a form is similar to that of the so-called hybrid inflation model [35] at the inflationary stage. Here, we only examine the inflationary stage and consider that the graceful exit from inflation occurs due to some additional mechanism. It follows from the relations in (2.2) and Eq. (3.14) that

$$\epsilon = \frac{\gamma_n^2}{2} \tanh^2(\gamma_n \kappa \phi) \approx \frac{\gamma_n^2}{2} \tanh^2[\gamma_n \kappa \phi_f \exp(N)], \quad (3.15)$$

$$\eta = \gamma_n^2, \quad (3.16)$$

$$\xi^2 = \gamma_n^4 \tanh^2 = 2\gamma_n^2 \epsilon \approx \gamma_n^4 \tanh^2[\gamma_n \kappa \phi_f \exp(N)]. \quad (3.17)$$

From Eq. (2.3), the equations in (2.4), and Eqs. (3.15)–(3.17), we obtain

$$\begin{aligned} n_s &\sim 1 + 2\gamma_n^2 - 3\gamma_n^2 \tanh^2(\gamma_n \kappa \phi) \\ &\approx 1 + 2\gamma_n^2 - 3\gamma_n^2 \tanh^2[\gamma_n \kappa \phi_f \exp(N)], \end{aligned} \quad (3.18)$$

$$r \sim 8\gamma_n^2 \tanh^2(\gamma_n \kappa \phi) \approx 8\gamma_n^2 \tanh^2[\gamma_n \kappa \phi_f \exp(N)], \quad (3.19)$$

$$\alpha_s \sim 6\gamma_n^4 \frac{\tanh^2(\gamma_n \kappa \phi)}{\cosh^2(\gamma_n \kappa \phi)} \approx 6\gamma_n^4 \frac{\tanh^2[\gamma_n \kappa \phi_f \exp(N)]}{\cosh^2[\gamma_n \kappa \phi_f \exp(N)]}. \quad (3.20)$$

For example, when $(N, \gamma_n, \kappa \phi_f) = (50, 3.00 \times 10^{-2}, 1.04 \times 10^{-20})$ and $(60, 4.00 \times 10^{-2}, 3.01 \times 10^{-25})$, we find $(n_s, r, \alpha_s) = (0.999, 6.17 \times 10^{-3}, 5.98 \times 10^{-7})$ and $(0.999, 9.90 \times 10^{-3}, 2.69 \times 10^{-6})$, respectively. For the latter case, the value of r becomes about $\mathcal{O}(0.01)$, although it is smaller than the BICEP by one order.

We finally mention that when equations in (2.8) can be solved with respect to H , we can explicitly get the corresponding scalar field theory. Indeed, however, this is too complicated calculation to be executed. That is why the investigations are limited here by presenting several simple examples.

4. Summary

In summary, we have reconstructed the scalar field models to explain the BICEP2 and Planck results by taking into consideration the running of the spectral index. As concrete examples, we have explored the chaotic inflation, natural (or axion) inflation, and a model of inflation in which the inflaton potential has a hyperbolic form.

Our method demonstrated in this Letter is considered to be an effective way of making scalar field models to account for any observational data taken from not only the current BICEP2 and Planck but also the future observational missions. Through this process, in principle, wider classes of the inflaton potentials which can realize inflation compatible with observations could be built. Consequently, it is expected that this procedure would lead to novel insights on the inflaton potentials to yield the consequences explaining any observations including the BICEP2 experiment and the Planck satellite. Furthermore, recently there have been raised some doubts on the prediction of the BICEP2 experiment. Even if this is true, our method gives the possibility to reconstruct scalar field

theories realizing viable inflation from the known values of slow-roll inflationary parameters. With release of more observational results, this reconstruction method may give more exact theoretical models of inflation.

In addition, very recently, an attempt of building $F(R)$ gravity theories [1] with inflation from observational data has been executed in Refs. [36,37] (see also Ref. [10]). As our next step, the formulations of reconstruction procedures of modified gravity theories such as $F(R)$ gravity consistent with the BICEP2 and Planck data will be developed in the future continuous work.

Acknowledgements

The work has been supported in part by Ministry of Education and Science of the Russian Federation, project TSPU-139 (S.D.O.), the JSPS Grant-in-Aid for Scientific Research (S) # 2224003 and (C) # 23540296 (S.N.), and that for Young Scientists (B) # 25800136 (K.B.).

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